Optimized Transformer Design: Inclusive of High-Frequency Effects

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***Abstract*— Switching circuits, operating at high frequencies, have led to considerable reductions in the size of magnetic components and power supplies. Nonsinusoidal voltage and current waveforms and high-frequency skin and proximity effects contribute to transformer losses. Traditionally, transformer design has been based on sinusoidal voltage and current waveforms operating at low frequencies. The physical and electrical properties of the transformer form the basis of a new design methodology while taking full account of the current and voltage waveforms and high-frequency effects. Core selection is based on the optimum throughput of energy with minimum losses. The optimum core is found directly from the transformer specifications: frequency, power output, and temperature rise. The design methodology is illustrated with a detailed design of a push–pull converter.**

***Index Terms*—High-frequency effects, magnetic circuits, optimization, switching circuits, transformers.**

NOMENCLATURE

A_c	Physical cross-sectional area of magnetic circuit.
A_m	Effective cross-sectional area of magnetic circuit.
A_p	Window area, $W_a \times$ cross-sectional area, A_c.
A_t	Surface area of wound transformer.
A_w	Bare wire conduction area.
B_m	Maximum flux density.
B_o	Optimum flux density.
$B_{\rm sat}$	Saturation flux density.
d	Thickness of foil or layer.
D	Duty cycle.
f	Frequency in hertz.
h	Coefficient of heat transfer by convection.
J	Current density.
K	Waveform factor [see (4)].
K_c	Material parameter [see (13)].
K_o, K_t, K_j	1.54 $\times$ 10^{-7}, 53.9 $\times$ 10^3, and 81.4 $\times$ 10^6 [see (29), (32), and (37)].
k_a, k_c, k_w	Dimensionless constants [see (26)].
k_f	Core stacking factor A_m/A_c.
k_p	Power factor.
k_s	Skin-effect factor $R_{\rm ac}/R_{\rm dc}$.
k_u	Window utilization factor W_c/W_a.
k_x	Proximity-effect factor $R'_{\rm ac}/R_{\rm dc}$.
m	Mass of core.
n	Number of windings.
N	Number of turns.
$P_{\rm fe}$	Iron losses.
$P_{\rm cu}$	Copper losses.
$R_{\rm dc}$	DC resistance of a winding.
$R_{\rm ac}$	AC resistance of a winding.
R_θ	Thermal resistance.
$T_{\rm max}$	Maximum operating temperature.
V_c	Volume of core.
V_w	Volume of windings.
VA	Volts–amp rating of winding.
$\langle v \rangle$	Average value of voltage over time τ.
W_a	Window area.
W_c	Electrical conduction area.
α, β	Material constants [see (13)].
δ	Skin depth.
Δ	d/δ.
ΔT	Temperature rise.
ρ_c	Mass density of core material.
ρ_w	Electrical resistivity of winding at $T_{\rm max}$.
τ	Time for flux to go from zero to B_m.
μ_r	Relative permeability of core material.
μ_o	Permeability of free space, $4\pi \times 10^{-7}$ H/m.

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I. INTRODUCTION

THE UNRELENTING movement to higher density integrated circuits continues unabated. Reductions in the size of magnetic components have been achieved by operating at higher frequencies, mainly in switching circuits. Traditionally, transformer design has been based on power frequency transformers with sinusoidal excitation. Empirical rules have evolved which generally lead to conservative designs. Nonsinusoidal excitation at high frequencies introduces new design issues: skin and proximity effects in windings and increased eddy current and hysteresis losses in cores. The starting point for an optimized design is the assumption that winding losses are approximately equal to the core losses [1], [2]. However, in a typical power frequency transformer, the ratio may be as high as 5 : 1. This is due to the fact that the flux density is limited by its saturation value. At the high end of the frequency scale, the transformer may be operating with a maximum flux density well below its saturation value to achieve an optimum design.

Failure mechanisms in magnetic components are almost always due to excessive temperature rise which means that the design must satisfy electrical and thermal criteria. A robust design must be based on sound knowledge of circuit analysis, electromagnetism, and heat transfer. This paper shows that familiar transformer equations, based on sinusoidal excitation conditions, may be restated to include the types of waveforms found in switching circuits. The analysis is based on fundamental principles. Approximations based on dimensional analysis are introduced to simplify calculations without compromising the generality of the design methodology or the underlying fundamental principles—a proliferation of design factors is avoided to retain clarity.

The purpose of a transformer is to transfer energy from the input to the output through the magnetic field. The aim is to optimize this energy transfer in a given application. The amount of energy transferred in a transformer is determined by the operating temperature, frequency, and flux density. This paper shows that for any transformer core there is a critical frequency. Above this critical frequency the losses can be minimized by selecting a flux density, which is less than the saturation flux density. Below the critical frequency, the throughput of energy is restricted by the limitation that flux density cannot be greater than the saturation value for the core material in question.

The primary objective of this paper is to establish a robust method which leads to optimized core selection and winding selection from the design specifications: power output, frequency, and temperature rise. Once the physical properties of the core and winding are established, detailed thermal and electrical models can be evaluated.

II. Minimizing the Losses

A. The Voltage Equation

Faraday's Law relates the impressed voltage on a winding v to the rate of change of flux density B

$$v = -NA_m \frac{dB}{dt} \tag{1}$$

where N is the number of turns and A_m is the effective cross-sectional area of the magnetic core. In the case of laminated and tape-wound cores, this is less than the physical area A_c due to interlamination space and insulation. The layout of a typical transformer is shown in Fig. 1 and the physical parameters are illustrated. The two areas are related by the core stacking factor k_f ($A_m = k_f A_c$). Typically, k_f is 0.95 for laminated cores.

Integrating (1) between the point where the flux density is zero and its maximum value (B_m) gives

$$\langle v \rangle = \frac{1}{\tau}\int_0^{\tau} v\,dt = \frac{1}{\tau} NB_m A_m \tag{2}$$

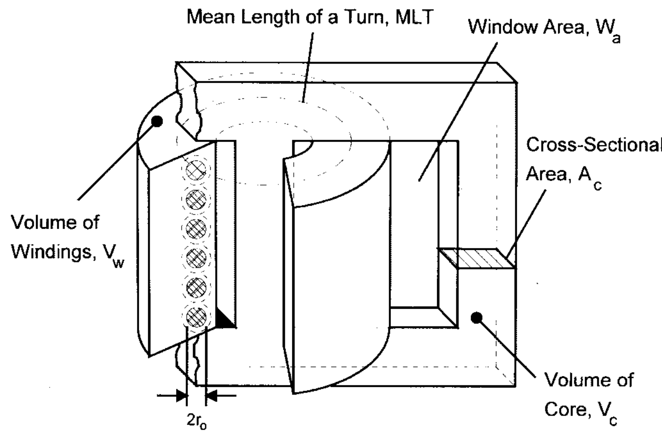

Fig. 1. Typical layout of a transformer.

where $\langle v \rangle$ is the average value of the impressed voltage in the time period τ. The form factor k is defined as the ratio of the rms value of the applied voltage waveform to $\langle v \rangle$

$$k = \frac{V_{\rm rms}}{\langle v \rangle}. \tag{3}$$

Combining (2) and (3) yields

$$V_{\rm rms} = \frac{k}{\tau/T} fNB_m A_m = KfNB_m A_m \tag{4}$$

where $f = \frac{1}{T}$ is the frequency of v and T is the period of v. Equation (4) is the classic equation for voltage in a transformer winding with K, the waveform factor, defined by k, τ, and T. Evidently, for a sinusoidal waveform $K = 4.44$, and for a square waveform $K = 4.0$. The calculation of K for the push–pull converter is illustrated in Section IV.

B. The Power Equation

Equation (4) applies to each winding of the transformer. Taking the sum of the volts–amp (VA) products for each winding of an n winding transformer

$$\sum \text{VA} = KfB_m A_m \sum_{i=1}^{n} N_i I_i. \tag{5}$$

The window utilization factor k_u is the ratio of the total conduction area W_c to the total window area W_a and therefore

$$\sum_{i=1}^{n} N_i A_{wi} = k_u W_a. \tag{6}$$

The current density in each winding is $J_i = I_i/A_{wi}$, where A_{wi} is the wire conduction area. Normally, the wire area and conduction area are taken as the area of bare conductor, however, we can account for skin effect in a conductor and proximity effect between conductors by noting that the increase in resistance due to these effects is manifested by reducing the effective conduction area. The skin-effect factor k_s is the increase in resistance (or decrease in conduction area) due to skin effect and likewise for the proximity-effect factor k_x

$$k_s = \frac{R_{\mathrm{ac}}}{R_{\mathrm{dc}}} \tag{7a}$$

$$k_x = \frac{R'_{\mathrm{ac}}}{R_{\mathrm{dc}}} \tag{7b}$$

$$k_u = \frac{k_b}{k_s k_x} \tag{8a}$$

$$W_c = k_u W_a \tag{8b}$$

where k_b is the ratio of bare conductor area to the window area. Typically, $k_b = 0.7$, $k_s = 1.3$, and $k_x = 1.3$ giving $k_u = 0.4$.

Combining (5) and (6) with the same current density J in each winding

$$\sum \mathrm{VA} = K f B_m k_f A_c J k_u W_a. \tag{9}$$

The product $A_c W_a$ appears in (9) and is an indication of the core size and is designated A_p. Rewriting (9) yields

$$\sum \mathrm{VA} = K f B_m J k_f k_u A_p. \tag{10}$$

C. Winding Losses

The total resistive losses for all the windings are

$$P_{\mathrm{cu}} = \sum R I^2 = \rho_w \sum_{i=1}^{n} \frac{N_i \mathrm{MLT}(J A_{wi})^2}{A_{wi}} \tag{11}$$

where ρ_w is the resistivity of the winding conductor and MLT is the mean length of a turn in the windings. Incorporating the definition of window utilization factor k_u (8a) and noting that the volume of the windings is $V_w = \mathrm{MLT} \times W_a$ and the conduction volume is $V_w \times k_u$, then

$$P_{\mathrm{cu}} = \rho_w V_w k_u J^2. \tag{12}$$

D. Core Losses

In general, losses are given in W/kg so that for a core of mass m $(m = \rho_c V_c)$

$$P_{\mathrm{fe}} = m K_c f^\alpha B_m^\beta = \rho_c V_c K_c f^\alpha B_m^\beta \tag{13}$$

where ρ_c is the mass density of the core material, V_c is the core volume, and K_c, α, and β are constants which can be established from manufacturer's data. Typical values are given in Table IV. The losses include hysteresis and eddy current losses. However, the manufacturer's data is normally measured for sinusoidal excitation. Furthermore, the test specimen size may be different from the designed component. Losses are dependent on the size of the core. In the absence of test data on the design core, the manufacturer's data must be used in establishing the constants in (13).

E. The Thermal Equation

The combined losses in the windings and core must be dissipated through the surface of the wound transformer. This topic is discussed in detail in [3]. The dominant heat-transfer mechanism is by convection. Newton's equation of convection relates heat flow to temperature rise (ΔT), surface area (A_t), and the coefficient of heat transfer h by

$$P = h A_t \Delta T \tag{14}$$

where P is the sum of the winding losses and the core losses.

Reference [3] separates the contributions from the winding and the core. The thermal resistance R_θ is the inverse of the product (hA_t) given by

$$\Delta T = R_\theta P. \tag{15}$$

The thermal resistance path for the winding losses $R_{\theta \mathrm{cu}}$ is in parallel with the resistance path of the core losses $R_{\theta \mathrm{fe}}$. Using the electrical analogy, the equivalent thermal resistance is

$$\frac{1}{R_\theta} = \frac{1}{R_{\theta \mathrm{cu}}} + \frac{1}{R_{\theta \mathrm{fe}}} = h A_t \tag{16}$$

where h and A_t are the equivalent values for the transformer treated as a single unit. For natural heat convection, h is a function of the height H of the transformer [4]

$$h = 1.42 \left[\frac{\Delta T}{H}\right]^{0.25}. \tag{17}$$

For an ETD44 core, $H = 0.045$ m and $h = 8.2$ W/m^2°C for a 50°C temperature rise. Evidently, the position of the transformer relative to other components will have a profound effect on the value of h. In fact, the value of h is probably the most uncertain parameter in the entire design. However, the typical value of $h = 10$ W/m^2°C is confirmed by test results in [1] and [5] for cores encountered in switching power supplies.

F. Optimization

Eliminating the current density in (12) using (10) yields

$$P_{\mathrm{cu}} = \rho_w V_w k_u \left[\frac{\sum \mathrm{VA}}{K f B_m k_f k_u A_p}\right]^2 = \frac{a}{f^2 B_m^2}. \tag{18}$$

Rewriting (13)

$$P_{\mathrm{fe}} = \rho_c V_c K_c f^\alpha B_m^\beta = b f^\alpha B_m^\beta. \tag{19}$$

The total losses are

$$P = \frac{a}{f^2 B_m^2} + b f^\alpha B_m^\beta. \tag{20}$$

The domain of P is in the first quadrant of the f-B_m plane. P is positive everywhere, and it is singular along the axes. If $\alpha = \beta$, P has a global minimum $\{\frac{dP}{d(fB_m)} = 0\}$ at

$$f_o B_o = \left[\frac{2a}{\beta b}\right]^{\frac{1}{\beta+2}}. \tag{21}$$

For $\alpha = \beta = 2$, with (18) and (19)

$$f_o B_o = \sqrt[4]{\frac{\rho_w V_w k_u}{\rho_c V_c K_c}} \sqrt{\frac{\sum \mathrm{VA}}{K k_f k_u A_p}}. \tag{22}$$

Given that B_o must be less the saturation flux density B_{sat}, there is a critical frequency, given by (22), above which the losses may be minimized by selecting an optimum value of flux density which is less than the saturation value $(B_o <$

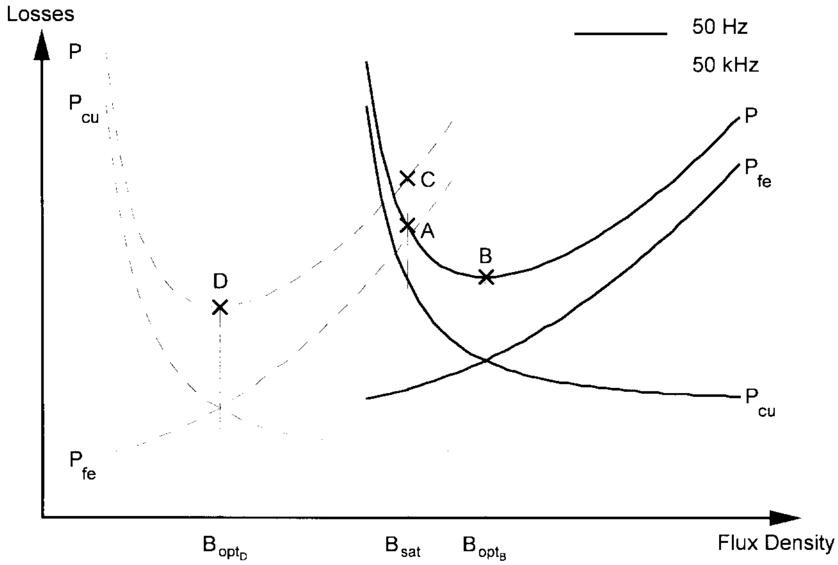

Fig. 2. Winding, core, and total losses at different frequencies.

B_{sat}). Equation (22) shows that f_oB_o is related to power density since A_p is related to core size.

In the more general case ($\alpha \neq \beta$), there is no global minimum. The minimum of P at any given frequency is obtained by taking the partial derivative with respect to B_m and setting it to zero

$$\frac{\partial P}{\partial B_m} = -\frac{2a}{f^2 B_m^3} + \beta b f^\alpha B_m^{\beta-1} = 0.$$

The minimum losses occur when

$$P_{\text{cu}} = \frac{\beta}{2} P_{\text{fe}}, \qquad \text{for a fixed frequency } f. \tag{23}$$

The minimum of P at any given flux density is obtained by taking the partial derivative with respect to f and setting it to zero. The minimum losses occur when

$$P_{\text{cu}} = \frac{\alpha}{2} P_{\text{fe}}, \qquad \text{for a fixed flux density } B_m. \tag{24}$$

Evaluation of (24) at $B_o = B_{\text{sat}}$ gives the critical frequency above which the total losses are minimized by operating at an optimum value of flux density which is less than the saturation value ($B_o < B_{\text{sat}}$)

$$f_o^{\alpha+2} B_o^{\beta+2} = \frac{2}{\alpha} \frac{\rho_w V_w k_u}{\rho_c V_c K_c} \left[\frac{\sum \text{VA}}{K k_f k_u A_p} \right]^2. \tag{25}$$

The nature of (20) is illustrated in Fig. 2. The two sets of curves shown are for low frequency (50 Hz) and high frequency (50 kHz).

At 50 Hz, the optimum flux density (at point B) is greater than the saturation flux density, and, therefore, the minimum losses achievable are at point A. However, the winding and core losses are not equal. At 50 kHz, the optimum flux density is less than the saturation flux density and the core and winding losses are equal. The first step in a design is to establish whether the optimum flux density given by the optimization criterion in (23) is greater or less than the saturation flux density as described in the next section.

III. The Design Equations

A. Dimensional Analysis

The physical quantities V_c, V_w, and A_t may be related to core size A_p by dimensional analysis

$$V_c = k_c A_p^{3/4} \tag{26a}$$

$$V_w = k_w A_p^{3/4} \tag{26b}$$

$$A_t = k_a A_p^{1/2}. \tag{26c}$$

All the coefficients are dimensionless. The values of k_c, k_w, and k_a vary for different types of cores [2], [6]. However, the combinations which are required for the transformer design are approximately constant. It may be stated that $k_a = 40$, $k_c = 5.6$, and $k_w = 10$. Further refinement of the dimensionless constants in (26) is somewhat redundant since the value of the heat-transfer coefficient h is not always well known. The pot core is sufficiently different in construction such that k_w is five. The calculation of A_p is described in the next section.

B. $B_o < B_{\text{sat}}$

The optimum design is at point D in Fig. 2. The optimum conditions established by (23) may be exploited to establish a formula for A_p in terms of the design specifications: output power, frequency, and temperature rise

$$P_{\text{cu}} P_{\text{fe}} = \frac{2\beta}{(\beta+2)^2} P^2. \tag{27}$$

P_{cu}, P_{fe}, and P are given by (18), (13), and (14), respectively. Taking $\beta = 2$ and invoking the dimensional analysis of (26) with rearrangement yields

$$A_p = K_o \left[\frac{\sum \text{VA}}{K f \Delta T} \right]^{4/3} [\rho_c K_c f^\alpha]^{2/3} \tag{28}$$

where

$$K_o = \left[\frac{4\rho_w}{k_f^2 k_u h^2} \frac{k_c k_w}{k_a^2} \right]^{2/3}. \tag{29}$$

Taking typical values: $\rho_w = 1.72 \times 10^{-8}$ Ω-m, $h = 10$ W/m^2°C, $k_a = 40$, $k_c = 5.6$, $k_w = 10$, $k_f = 1.0$, and $k_u = 0.4$ yields $K_o = 1.54 \times 10^{-7}$. Units of the International System have been used, and A_p is in m^4.

The optimum value of current density in the windings may be found from the optimum criterion (23) using the equations for copper losses (12) and thermal heat transfer (14)

$$J_o = \sqrt{\frac{\beta}{\beta+2}\frac{hA_t\Delta T}{\rho_w V_w k_u}}. \tag{30}$$

Employing the dimensional analysis (26) with $\beta = 2$ in (30) yields

$$J_o = K_t\sqrt{\frac{\Delta T}{A_p^{1/4}}} \tag{31}$$

where

$$K_t = \sqrt{\frac{h}{2\rho_w k_u}\frac{k_a}{k_w}}. \tag{32}$$

Taking typical values: $\rho_w = 1.72 \times 10^{-8}$ Ω-m, $h = 10$ W/m^2°C, $k_a = 40$, $k_w = 10$, and $k_u = 0.4$ gives $K_t = 53.9 \times 10^3$.

The flux density is found from the power equation (10)

$$B_m = \frac{\sum \text{VA}}{Kfk_f k_u J A_p}. \tag{33}$$

Substituting the optimum value of J given by (31) and A_p given by (28) into (33) results in an expression for the optimum flux density

$$B_o = \frac{1}{K_o^{7/8}K_t}\frac{\sqrt{\Delta T}}{k_f k_u}\left[\frac{Kf\Delta T}{\sum \text{VA}}\right]^{1/6}\frac{1}{[\rho_c K_c f^\alpha]^{7/12}}. \tag{34}$$

Evidently, (28) and (34) may be evaluated from the specifications of the application and the material constants.

C. $B_o > B_{\text{sat}}$

In this case, the saturation of the magnetic material dictates that the design is at point A in Fig. 2. The value of B_m in the voltage equation is fixed by B_{sat}. The current density is found by combining the winding losses (12) and the core losses (13) and using the thermal equation (14)

$$J^2 = \frac{hA_t\Delta T}{\rho_w V_w k_u} - \frac{\rho_c V_c K_c f^\alpha B_m^\beta}{\rho_w V_w k_u}. \tag{35}$$

Despite the unwieldy nature of (35), J may be expressed in terms of the transformer specifications with the aid of dimensional analysis equations as follows:

$$J = \sqrt{\frac{2K_t^2\Delta T}{A_p^{1/4}} - K_j\rho_c K_c f^\alpha B_m^\beta} \tag{36}$$

where

$$K_j = \frac{k_c}{\rho_w k_u k_w}. \tag{37}$$

Taking typical values: $\rho_w = 1.72 \times 10^{-8}$ Ω-m, $k_c = 5.6$, $k_w = 10$, and $k_u = 0.4$ gives $K_j = 81.4 \times 10^6$.

J is a function of A_p in (36), which has yet to be calculated, and, therefore, some iteration is required. Evidently, substituting for J given by (10) in (36) yields a second-order polynomial in A_p, where

$$f(A_p) = a_0A_p^2 - a_1A_p^{7/4} + a_2 = 0 \tag{38}$$

and

$$a_0 = K_j\rho_c K_c f^\alpha B_m^\beta$$
$$a_1 = 2K_t^2\Delta T$$
$$a_2 = \left[\frac{\sum \text{VA}}{KfB_m k_f k_u}\right]^2.$$

The roots of $f(A_p)$ are found numerically using the Newton Raphson method

$$A_{p_{i+1}} = A_{p_i} - \frac{f(A_{p_i})}{f'(A_{p_i})} = A_{p_i} - \frac{a_0A_{p_i}^2 - a_1A_{p_i}^{7/4} + a_2}{2a_0A_{p_i} - \frac{7}{4}a_1A_{p_i}^{3/4}}. \tag{39}$$

Normally one iteration is sufficient.

The initial estimate of A_p is found by assuming the total losses are equal to twice the copper losses (at point A in Fig. 2, the total losses are less than twice the copper losses). In this case, J is given by (31) and may be substituted into the power equation (10) to give

$$A_{p_i} = \left[\frac{\sum \text{VA}}{KfB_m k_f k_u K_t\sqrt{\Delta T}}\right]^{8/7}. \tag{40}$$

D. Design Methodology

The overall design methodology is shown in flowchart form in Fig. 3. The core manufacturer normally supplies the core data: cross section A_m, window area W_a, the mean length of a turn MLT ($V_w = \text{MLT} \times W_a$), and the core mass m ($m = \rho_c V_c$). The number of turns in each winding is found from (4)

$$N = \frac{V_{\text{rms}}}{KfB_m A_m}. \tag{41}$$

In this equation, B_m is interpreted as B_o or B_{sat} depending on which is lower. The selected core from standard designs may not correspond exactly to the optimum selection, and, therefore, the current density may be calculated using (35). With $h = 10$ W/m^2°C and $k_a = 40$, then

$$J = \sqrt{\frac{400\sqrt{A_p}\Delta T - mK_c f^\alpha B_m^\beta}{\rho_w k_u V_w}}. \tag{42}$$

The resistivity of the conductor at the maximum operating temperature is given by

$$\rho_w = \rho_{20}[1 + \alpha_{20}(T_{\max} - 20°\text{C})] \tag{43}$$

where $T_{\max}$ is the maximum temperature, ρ_{20} is the resistivity at 20°C, and α_{20} is the temperature coefficient of resistivity at 20°C. The wire sizes are selected from standard wire tables, which normally specify resistance in Ω/m at 20°C. The winding loss is then

$$P_{\text{cu}} = \text{MLT} \times N \times (\Omega/\text{m}) \times [1 + \alpha_{20}(T_{\max} - 20°\text{C})] \times I^2. \tag{44}$$

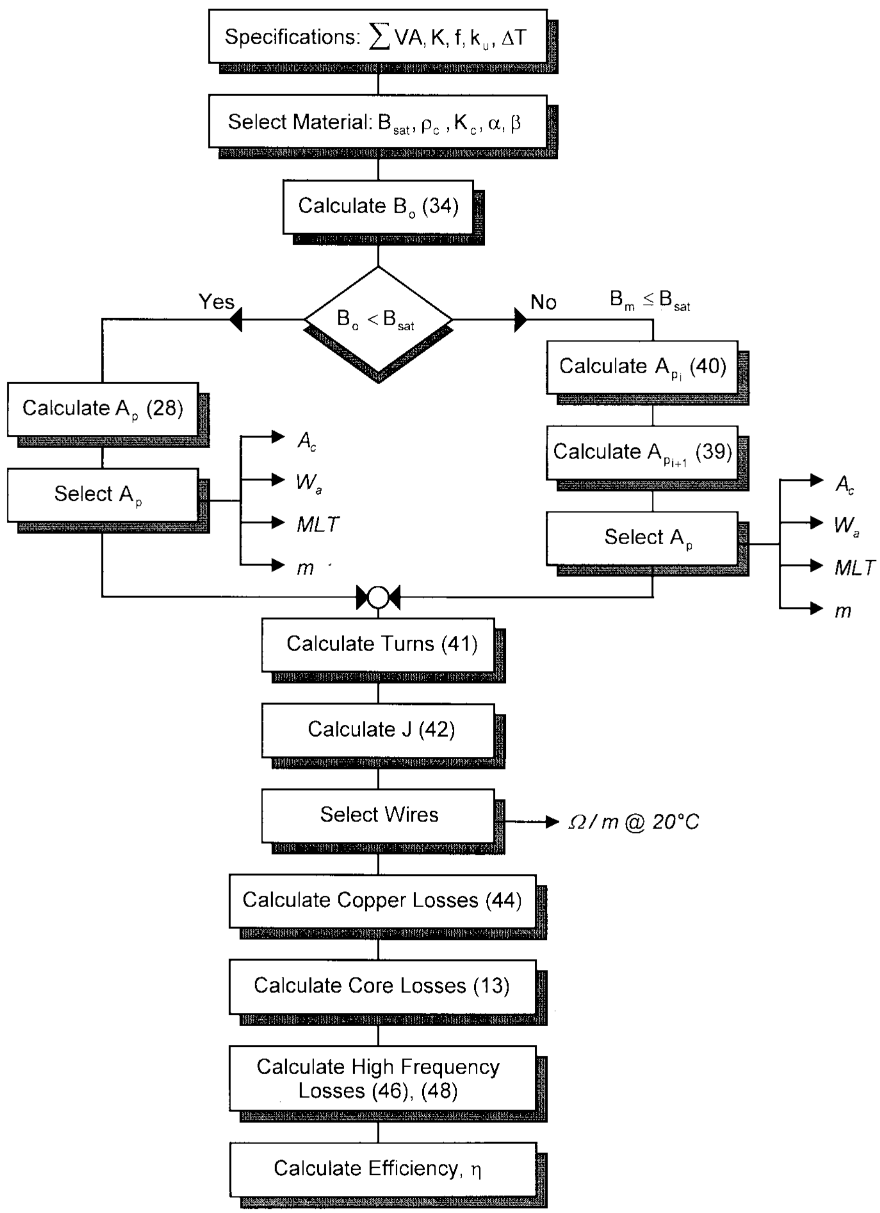

Fig. 3. Flowchart of design process.

E. Skin and Proximity Effects

An isolated round conductor carrying ac current generates a concentric alternating magnetic field which, in turn, induces eddy currents (Faraday's Law). The internal impedance of a round conductor of radius r_o is [7]

$$Z_i = R_{\mathrm{dc}} \frac{m r_o I_0(m r_o)}{2 I_1(m r_o)} \tag{45}$$

where

$$m = (1+j)/\delta.$$

δ is called the skin depth, $\delta = \sqrt{\rho_w/\pi f \mu_0}$, I_0, and I_1 are modified Bessel functions of the first kind, and j is the complex operator $j = \sqrt{-1}$. The ac resistance of a round conductor is given by the real part of Z_i, and internal inductive reactance is given by the imaginary part of Z_i. This expression is too cumbersome to use and the following approximations are useful: recalling $k_s = R_{\mathrm{ac}}/R_{\mathrm{dc}}$

$$\begin{aligned} k_s &= 1 + \frac{(r_o/\delta)^4}{48 + 0.8(r_o/\delta)^4} && \cdots\ r_o/\delta < 1.7 \\ &= 0.25 + 0.5\left(\frac{r_o}{\delta}\right) + \frac{3}{32}\left(\frac{\delta}{r_o}\right) && \cdots\ r_o/\delta > 1.7. \end{aligned} \tag{46}$$

The proximity-effect factor for p layers is given by Dowell [8]

$$k_x = \Delta \left\{ \frac{\mathrm{Sinh}2\Delta + \mathrm{Sin}2\Delta}{\mathrm{Cosh}2\Delta - \mathrm{Cos}2\Delta} + \frac{2(p^2-1)}{3} \frac{\mathrm{Sinh}\Delta - \mathrm{Sin}\Delta}{\mathrm{Cosh}\Delta + \mathrm{Cos}\Delta} \right\}. \tag{47}$$

A simple approximation is [9]

$$k_x = 1 + \frac{5p^2 - 1}{45} \Delta^4 \tag{48}$$

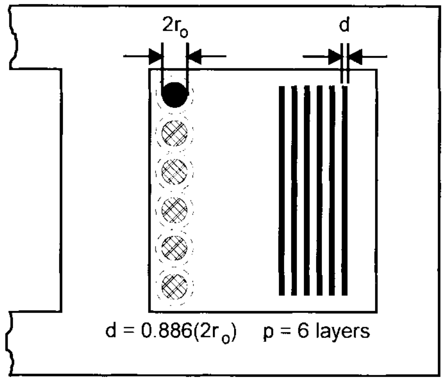

Fig. 4. Winding layout.

TABLE I
SPECIFICATIONS

Output	24 V, 12.5 A
Input	36 → 72 V
Frequency, f	50 kHz
Temperature Rise, ΔT	30 °C
Ambient Temperature, T_a	45 °C
Efficiency, η	90 %

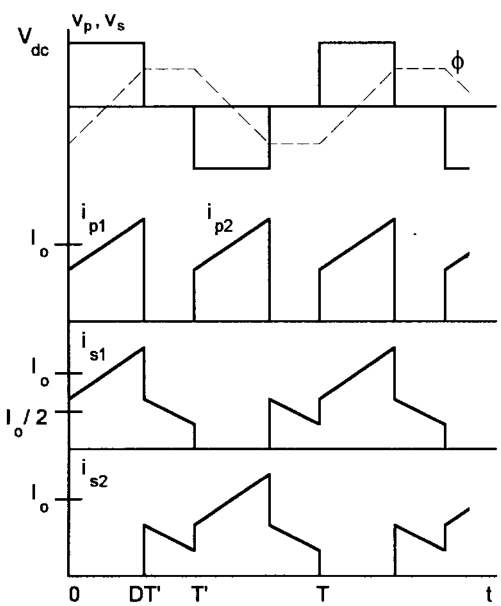

Fig. 5. Push–pull converter circuit.

where Δ is the ratio of the thickness of a layer of foil d to the skin depth δ (see Fig. 4).

IV. DESIGN EXAMPLE: PUSH–PULL CONVERTER

The example which follows is based on a push–pull converter with the specifications given in Table I.

The push–pull converter and its associated voltage and current waveforms are shown in Figs. 5 and 6. We assume for simplicity that the turns ratio is 1 : 1.

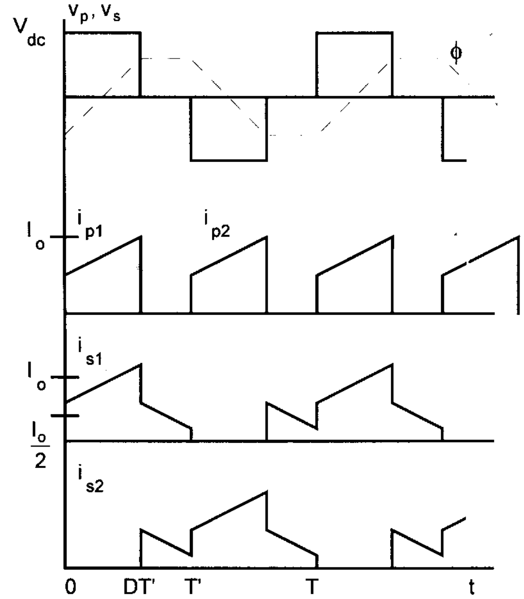

Fig. 6. Push–pull converter waveforms.

TABLE II
SPECIFICATIONS

K_c	1.9×10^{-3}
α	1.24
β	2.0
ρ_m	4800 kg/m^3
B_{sat}	0.4 T

TABLE III
CORE AND WINDING SPECIFICATIONS

A_c	1.73 cm^2
W_a	2.78 cm^2
A_p	4.81 cm^4
m	0.085 kg
k_f	1.0
k_u	0.4
MLT	7.77 cm
ρ_{20}	1.72 μΩ-cm
α_{20}	0.00393

In Fig. 6, switch 1 turns on at $t = 0$ and turns off at time DT'. By defining the duty cycle in this manner, the combined on time of the two switches is DT and the output voltage is DV_{dc}. The rms value of the applied voltage waveform on each winding is $\sqrt{D}V_{\mathrm{dc}}$. The dc value over the time period when the flux is being established is V_{dc}. The switching period is T and each switch controls the voltage waveform for $T' = T/2$.

TABLE IV
TYPICAL CORE DATA

Material	Sat. Flux Density	Relative Permeability	Resistivity (Ω-m)	ρ_c kg/m^3	K_c	α	β
Powder Iron	2.1	4500	0.01	6000	0.1 → 10	1.1	2.0
Si-Steel	2.0	10000	0.01	7650	0.5×10^{-3}	1.7	1.9
Ni-Mo Alloy	0.8	250	0.01	13000	5.0×10^{-3}	1.2	2.2
Ferrite (Mn-Zn)	0.4	2000	1.0	4800	1.9×10^{-3}	1.24	2.0
Ferrite (Ni-Zn)	0.3	400	1000	4800	2.5×10^{-5}	1.6	2.3
Metallic Glass	1.6	10000	10^{10}	60000			

Values are typical and are given for comparison purposes only. Specific values should be established from manufacturer's data sheets for specific cores. B_m in tesla and f in hertz, m in kg yields watts in (13) with K_c above.

From Section II-A

$$k = \frac{V_{\rm rms}}{\langle v\rangle} = \frac{\sqrt{D}V_{\rm dc}}{V_{\rm dc}} = \sqrt{D}$$
$$\frac{\tau}{T} = \frac{DT'/2}{T} = \frac{D}{4}$$
$$K = \frac{k}{\tau/T} = \frac{\sqrt{D}}{D/4} = \frac{4}{\sqrt{D}}.$$

For $D = 1.0$, $K = 4.0$ as expected for a square waveform.

An interesting feature of the push–pull circuit is that when both switches are off, the current circulates in the secondary windings. This circulating current contributes to heating, but there is no transfer of power through the transformer. Our definition of power factor must take this into account.

The rms value of the secondary current (neglecting the ripple), as shown in Fig. 6, is given by

$$I_s = \frac{I_o}{2}\sqrt{(1+D)}.$$

The rms value of the secondary voltage is $V_s = \sqrt{D}V_{\rm dc} = V_o/\sqrt{D}$.

The VA rating of each secondary winding is now

$$V_sI_s = \frac{1}{2}\frac{\sqrt{1+D}}{\sqrt{D}}V_oI_o = \frac{1}{2}\frac{\sqrt{1+D}}{\sqrt{D}}P_o.$$

Recalling the definition of power factor and noting that for each winding the average power $\langle p\rangle = P_o/2$, where P_o is the total output power, the power factor of each secondary winding is

$$k_{ps} = \frac{\langle p\rangle}{V_{\rm rms}I_{\rm rms}} = \sqrt{\frac{D}{1+D}}.$$

For $D = 1$, $k_{ps} = 1/\sqrt{2}$ as expected.

The rms values of the input voltage and current are

$$V_p = \sqrt{D}V_{\rm dc} \quad I_p = \sqrt{(D/2)}I_{\rm dc}.$$

For 1 : 1 turns ratio, $I_{\rm dc} = I_o$ and the power factor in each primary winding is then given by $k_{pp} = 1/\sqrt{2}$.

We can now sum the VA ratings over the two input windings and the two output windings. The average power through each secondary winding is $P_o/2$, and the average power through each primary winding is $P_o/(2\eta)$. Thus, we have

$$\sum {\rm VA} = \left(\frac{1}{\eta k_{pp}}\left(\frac{P_o}{2}+\frac{P_o}{2}\right) + \frac{1}{k_{ps}}\left(\frac{P_o}{2}+\frac{P_o}{2}\right)\right) = \left(\frac{\sqrt{2}}{\eta} + \sqrt{\frac{1+D}{D}}\right)P_o. \quad (49)$$

For the input voltage range, the duty cycle can vary between 33%–67%. For an input voltage of 36 V, the duty cycle is $24/36 = 67\%$. The waveform factor $K = 4/\sqrt{D} = 4.88$.

A. Step 1. Core Selection

Ferrite would normally be used for this type of application at the specified frequency. The material specifications for Siemens N67 Mn–Zn ferrite are listed in Table II.

The output power of the transformer is $P_o = (24+1.5) \times 12.5 = 318.8$ W, assuming a forward voltage drop of 1.5 V for the diode. The power factor and VA ratings of the winding are established above. In terms of core selection, the worst case occurs at maximum duty cycle, i.e., $D = 0.67$, $K = 4.88$, and from (49), $\sum$ VA $= 1005$ VA.

The optimum flux density (34) is $B_o = 0.112$ T. The optimum flux density is less than $B_{\rm sat}$, and A_p from (28) is 3.644 cm^4. The Siemens ETD44 E core is suitable. The core specifications are given in Table III.

B. Step 2. Turns

The ratio $V_{\rm rms}/K$ is evidently $V_o/4$ from the above analysis and therefore independent of D. The number of turns (41) in each primary is $N_p = 6.2$. Choose $N_p = 6$ turns and $N_s = 6$ turns.

C. Step 3. Wire Sizes

The maximum temperature is $T_{\max} = T_a + \Delta T = 45 + 30 = 75$°C. The current density (42) for the chosen core is $J = 2.644 \times 10^6$ A/m^2, based on the core and winding specifications in Table III.

For the primary windings with $D = 0.67$, the rms value of the voltage is $V_p = \sqrt{D}V_{\rm dc} = \sqrt{0.67}(36) = 29.5$ V. The current and wire size are

$$I_p = \frac{P_o/2}{\eta k_{pp} V_p} = \frac{318.8/2}{(0.9)(0.707)(29.5)} = 8.5 \text{ A}$$
$$A_w = I_p/J = 3.215 \text{ mm}^2.$$

Standard 0.1 × 30-mm copper foil with a dc resistance of 5.8 mΩ/m @ 20°C meets this requirement. For the secondary windings

$$I_s = \frac{I_o}{2}\sqrt{1+D} = \frac{12.5}{2}\sqrt{1+0.67} = 8.08 \text{ A}$$
$$A_w = I_s/J = 3.056 \text{ mm}^2.$$

Again, standard 0.1 × 30-mm copper foil meets this requirement.

D. Step 4. Copper Losses

For the primary windings, the losses are calculated with (44), $R_p = 3.3$ mΩ, and the copper losses in the two primary windings are $R_p I_p^2 \times 2 = 0.477$ W.

The secondary winding resistance is $R_p = 3.3$ mΩ. The copper losses in the two secondary windings are $R_s I_s^2 \times 2 = 0.431$ W.

E. Step 5. High-Frequency Effects

The skin depth in copper at 50 kHz is 0.295 mm, which is greater than the thickness of the foil and therefore does not present a problem. For $\Delta = 0.1/0.295$ and $p = 6$, k_x given by (47) is 1.05. A 0.1 × 30-mm foil is equivalent in area to a number 11 AWG round wire with $r_o = 1.154$ mm, and k_s given by (46) is 2.2 (see Fig. 4). With foil windings, the total losses become $0.908 \times 1.05 = 0.953$ W.

F. Step 6. Core Losses

The core losses (13) are $P_{\text{fe}} = 1.369$ W.

G. Step 7. Efficiency

The total losses are 2.322 W, and the efficiency η is 99.3%.

V. Conclusion

A new methodology for designing transformers has been described. The model takes account of switching-type waveforms encountered in switching-mode power supplies, inclusive of high-frequency skin and proximity effects. The selection of the core has been optimized to minimize the core and winding losses. Accurate approximations have been provided to facilitate calculations. The design procedure has been illustrated with the full design of a push–pull converter. The methodology is eminently suitable for use in a computer application in conjunction with a database of core materials.

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